Using a Stick and Brains How the Ancients Calculated the Size of the Earth

Man has been looking at the heavens since times immemorial. All the early civilizations such as the Babylonians, The Egyptians, Indians, and Chinese came up with their own conjecture of what they saw in the night sky. Most associated the heaven with the abode of gods or gods personified. It was considered sacrilegious to call the Stars, Sun, and Moon as inanimate bodies as they were gods, goddesses, and deities. For example the early Greeks thought the Sun to be a fiery chariot driven across the sky by the god Helios. The early Babylonians and Egyptians have mapped out the skies in their minds and assigned names and locations to various heavenly bodies. But their thinking was similar to the Greeks.

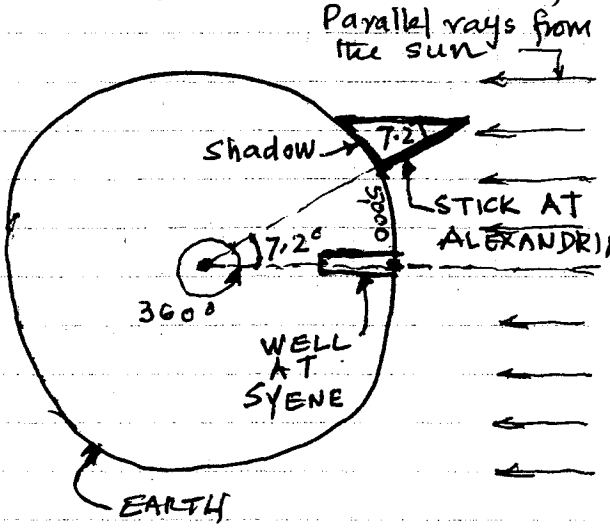
The Greeks were the first to move forward from these ancient concepts of objects in the sky from gods to something else. In the Greek world in the sixth century BC the society and intelligensia had developed enough tolerance to accept new radical ideas put forward by some natural philosophers away from the mythological concepts of gods. For example Anaximander (570BC) of Miletus argued that the sun was a hole in a fire-filled ring that encircled the earth and revolved around it. Similarly the moon and stars were nothing more than holes in the firmament revealing otherwise hidden fires. Others such as Xenophanes (540 BC) and Colophon believed that the earth exuded combustible gases that accumulated at night until they reached a critical mass and ignited thereby creating the sun. The night came when these gases burned out leaving behind few sparks which were the stars. Similarly the moon was gases burning over a twenty eight day cycle. These ideas though not scientific were far from mythological explanations and approaching science though not scientific.

Pythagoras (532BC) of Samos with his passion for mathematics and numbers was able to move these rationalist ideas towards more rigorous thinking and analysis. Though his ideas were more in terms of music but gave impetus to more rational thinking. Once he has shown the application of mathematics could be used to describe and explain music subsequent generation of scientists used numbers to explore everything from the trajectory of cannon ball to chaotic weather patterns.

The Greeks overtime had already established the spherical shape of earth and rudimentary concepts of gravity. Euclid's "Elements" was published with its geometrical formulas and solutions. Previous observers such as Anaxagoras (470BC) and Aristarchus had already established some facts regarding relative positions and distances of the Sun, Moon, and Earth. To calculate the sizes and distances of these bodies the size of earth, the circumference and the diameter were required. This feat was accomplished by Eratosthenes (276 BC) of Cyrene which is on the coast of modern day Libya.

Eratosthenes was for many years the chief librarian of the famous Library of Alexandria which was the most the most respected institution of learning in the world at that time and his position the most prestigious academic post in the ancient world. Cosmopolitan Alexandria has taken over from Athens as the intellectual hub of the Mediterranean. While at the library learned of a well in the town of Syene near modern day Aswan in Southern Egypt which is almost on the Tropic of Cancer. At noon on June 21 st each year the day of summer solstice the sun shown directly into the well and illuminated all the way to the bottom. This was an indication that the sun was directly over head and perpendicular to the plane of earth at that point. This never happened at Alexandria which was several hundred kilometers away. As shown in the attached he proceeded to exploit this to measure the circumference of the earth. The other calculations followed once the diameter of earth was know.

Circumference of Earth by Eratosthenes ($40{,}100\ km = 4.01 \times 10^4\ km$)



Eratosthenes used the shadow cast by a stick at Alexandria to calculate the circumference of earth. He conducted the experiment at the 'summer solstice, when ther Earth was at maximum tilt and towns such as Syene along the Tropic of cancer were closest to the sun and sun was directly overhead at those towns.

At the time sunlight was plunging straight down the well at Syene, Eratosthenes stuck a stick vertically in the ground at Alexandria and measured the angle between sun rays and the stick. By geometrical properties of circle this is equivalent to the angle between two radial lines drawn from Alexandria and Sayene to the center of the Earth. He measured the angle to be 7.2°

$\theta = 7.2^\circ \simeq 1/50$
$l = 5{,}000$ stades
Circumference $= 50 \times 5000$
$= 250{,}000$ stades
$\simeq 46{,}250$ km

Knowing that a circle is 360 thes represents $\frac{7.2}{360} \simeq \frac{1}{50}$ of the earth's circumference.
The arc distance between Alexandria and Sayene was approximately 5000 stades. This gave him circumference of of earth as $50 \times 5000 = 250{,}000$ stades.
A stade was equivalent to 185 meters which makes it 46,250 km. as compared to modern calculation of 40,100 km. The concuit calculation is only 15° bigger. If he had used Egyptian stade which was 157 meters the result would have been 39,250 km accurate to 2%. Finally it was generally accepted as 40,000 km at that time.

Diamter of the Earth (12,750 KM = 1.275×10^4 KM)

Once he knew the circumference the diameter was calculated using the well known formula $C^{ce} = 2\pi R = \pi D$

so the diameter came out to be = $40000 / 3.142 \cong 12,700$ KM

Diameter of The Moon (Lunar Eclipse) (3,480KM = 3.48×10^3 KM)

Eratosthenes deduced the size of the moon by observing the movement of moon in the earth's shadow during the Lunar Eclipse.

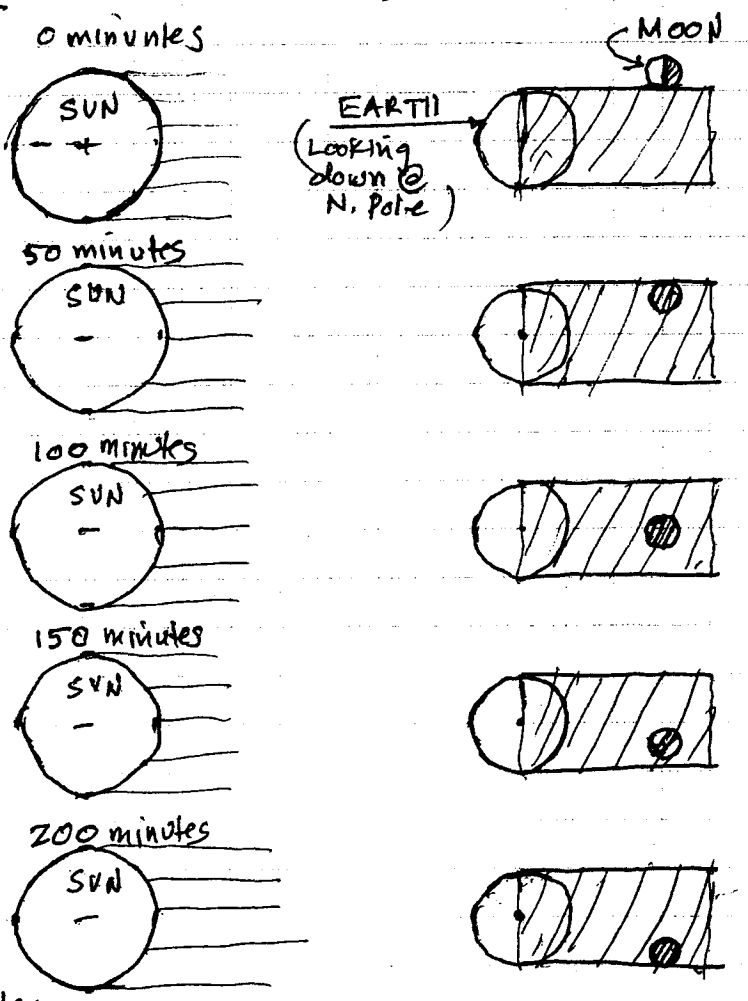
The diagram shows moon passing through the Earth's shadow.

it takes Moon 50 min to be covered completely by earth's shadow.

So 50 minutes indicate the diameter of moon.

The moon emerges from earths shadow in 200 minutes which indicates the size of earth's diameter

So the conclusion drawn was moon's diameter was 50/200 = 1/4 of earths diameter. which comes out to approximately 3200 km



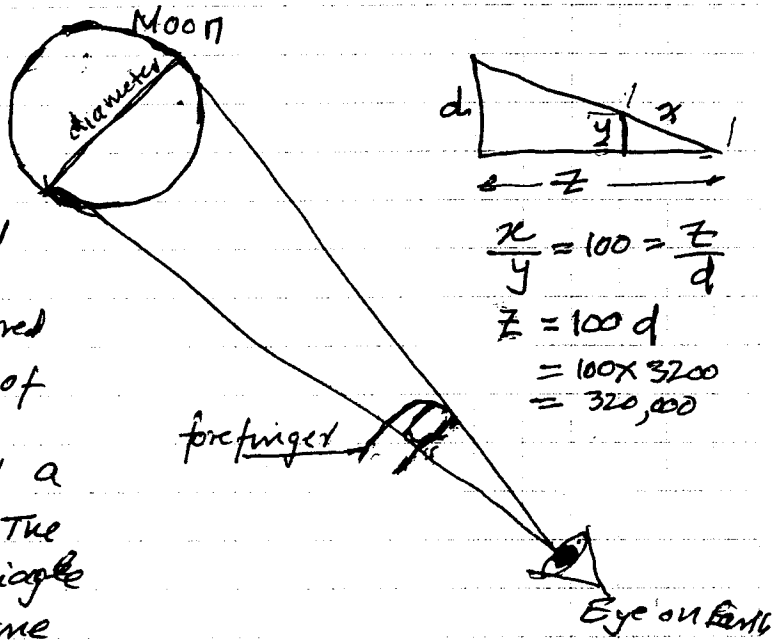
$\frac{50}{200} \simeq \frac{\text{diameter of moon}}{\text{diameter of earth}}$

$\simeq 1/4$

Diameter of moon $= \frac{1}{4} \times 12700$

$\cong 3200$ km

Distance of Earth from Moon ($384{,}000\ km = 3.84 \times 10^5\ km$)

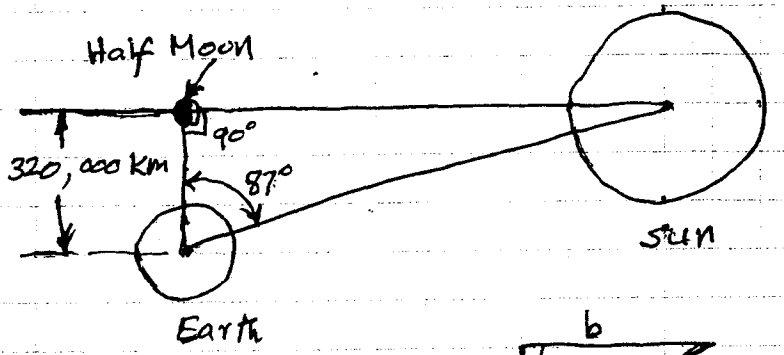


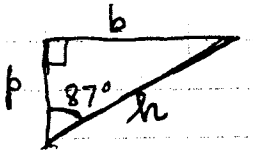
Eratosthenes came up with an unbelievably simple way to figure this out.
On a full moon he closed one eye and with his outstretched arm he covered the moon with the end of his forefinger.
The forefinger nail formed a triangle with the eye. The moon form a similar triangle with huge sizes but same proportions. He found the ratio between his fingernail height and length of arm as 100:1.
Using similar triangles he deduced the distance to the moon to be roughly 100 times the diameter, which came to $3200 \times 100 = 320{,}000$ km

Distance of Sun from Earth ($150{,}000{,}000\ KM = 1.5 \times 10^8\ Km$)

Angle 89.85°

In order to determine the distance of Sun from Earth Eratosthenes used previously established hypothesis by Anaxagoras of Clazomenae and an arguement by Aristarchus of Samos
In the third century BC, Aristarchus built on Anaxagoras idea if moon shine was reflected sunshine then half moon must occur when the Sun, Moon and Earth





$\cos 89.85° = \frac{p}{h}$

$.0025 = p/h$

$h = \frac{p}{.0025} \simeq 400\,p$

$\simeq \underline{400} \times$ moon dist

$\cos 87 = \frac{p}{h}$

$0.05 = \frac{p}{h}$

$h = \frac{p}{.05} \simeq 20\,p$

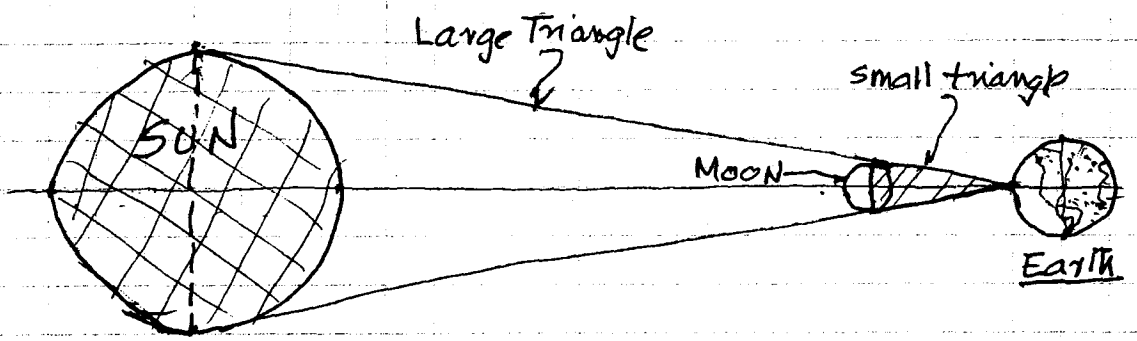
$= \underline{20} \times 320{,}000$ km

formed a right-angled triangle. He measured the angle to be 87° which by trignometry comes out to 20 times the distance to moon. Since the distance to moon was available by previous calculations the distance to sun was obtained. In fact the correct angle is 89.85° and sun is 400 times the distance of Moon.

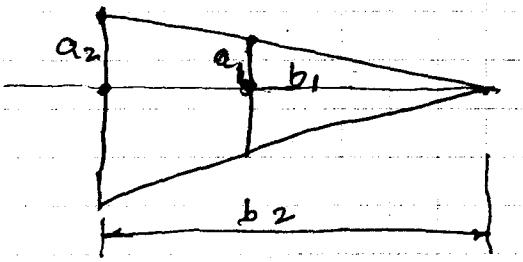
Note how much difference a slight change in angle from 87° to 89.85° degrees makes from 20 times to 400 times moon distance.

Size of the Sun – Diameter (1,390,000 km or 1.39×10^6 km)

Using the established fact that Moon fits almost perfectly over the Sun during a solar eclipse. Using principles of geometry of similar triangles the ratio of Sun's diameter to the Sun's distance from the Earth



must be the same as the ratio of Moon's diameter to the moon's distance from the Earth.



$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

Sun Radius $= a_2 = \dfrac{a_1 \times b_2}{b_1}$

$= \dfrac{\text{Moon Radius} \times \text{Moon Distance}}{\text{Sun Distance}} = \dfrac{1600 \times 150{,}000{,}000}{384{,}000}$

$= 625{,}000$ KM

Sun Diameter $= 2a_2 = 2 \times 625{,}000 \cong 1{,}250{,}000$ KM vs $1{,}390{,}000$ KM

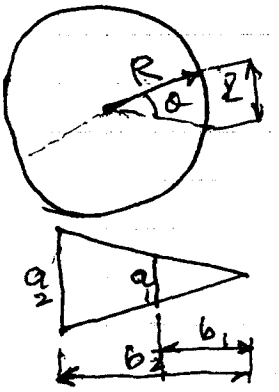
Geometrical Formulas Used;

Properties of Circle:

Circumference $= 2\pi R = \pi D$

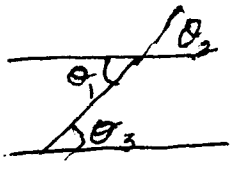
Arc Length $= R\theta$

Similar Triangles $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2}$



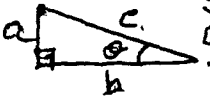
Angle between Parallel Lines

$\theta_1 = \theta_2 = \theta_3$



Right Angled Triangle (Pythagoras Theorem)

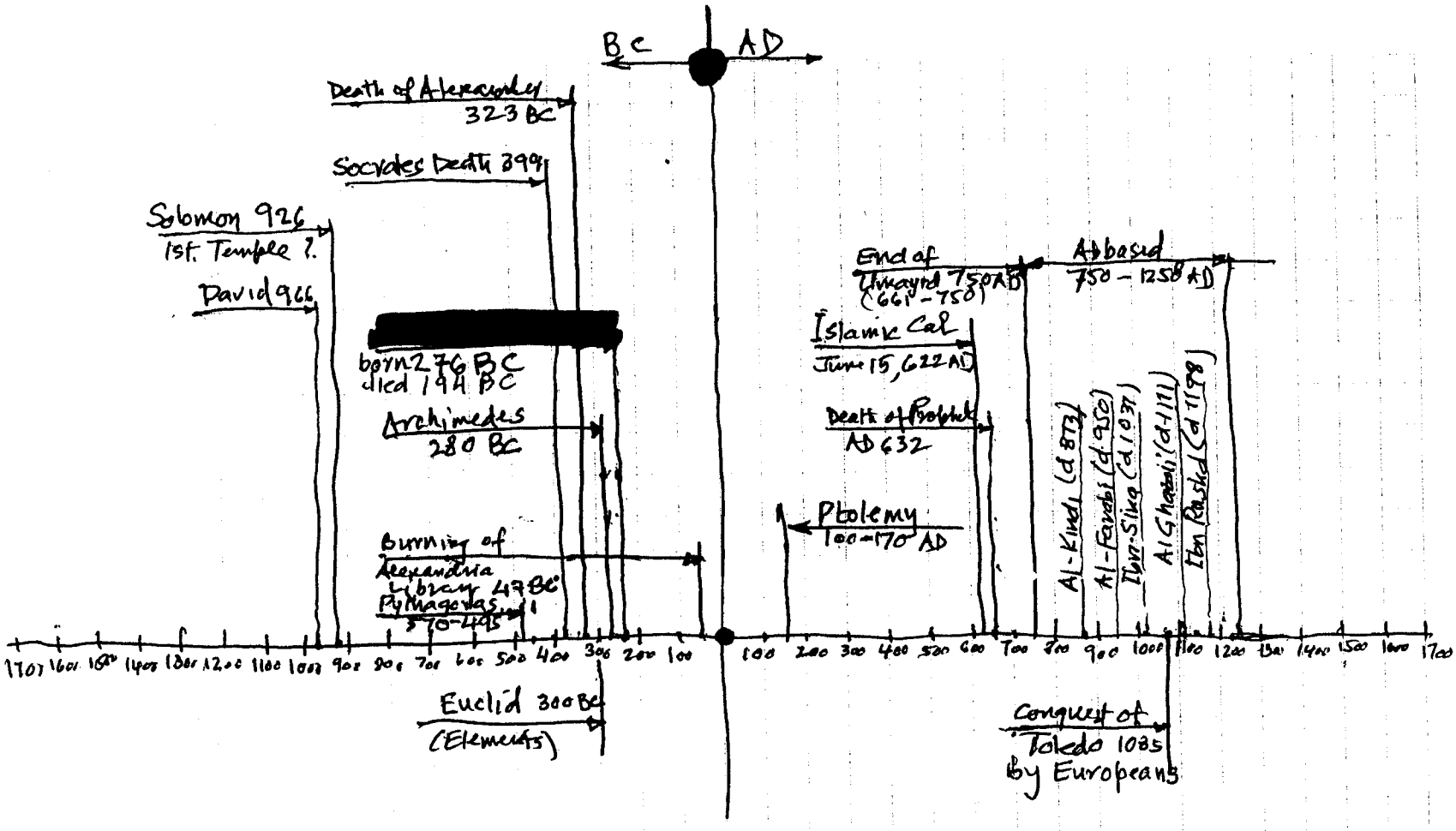
$a^2 + b^2 = c^2$



$\sin\theta = a/c$

$\cos\theta = b/c$

$\tan\theta = a/b$



Sources Used:

Big Bang - The Origin of The Universe by Simon Singh - Harper
Atlas of World History - Vol. I Hermann Kinder & Werner Hilgemann - Anchor Books
The History of Philosophy - A.C. Grayling. Penguin Press

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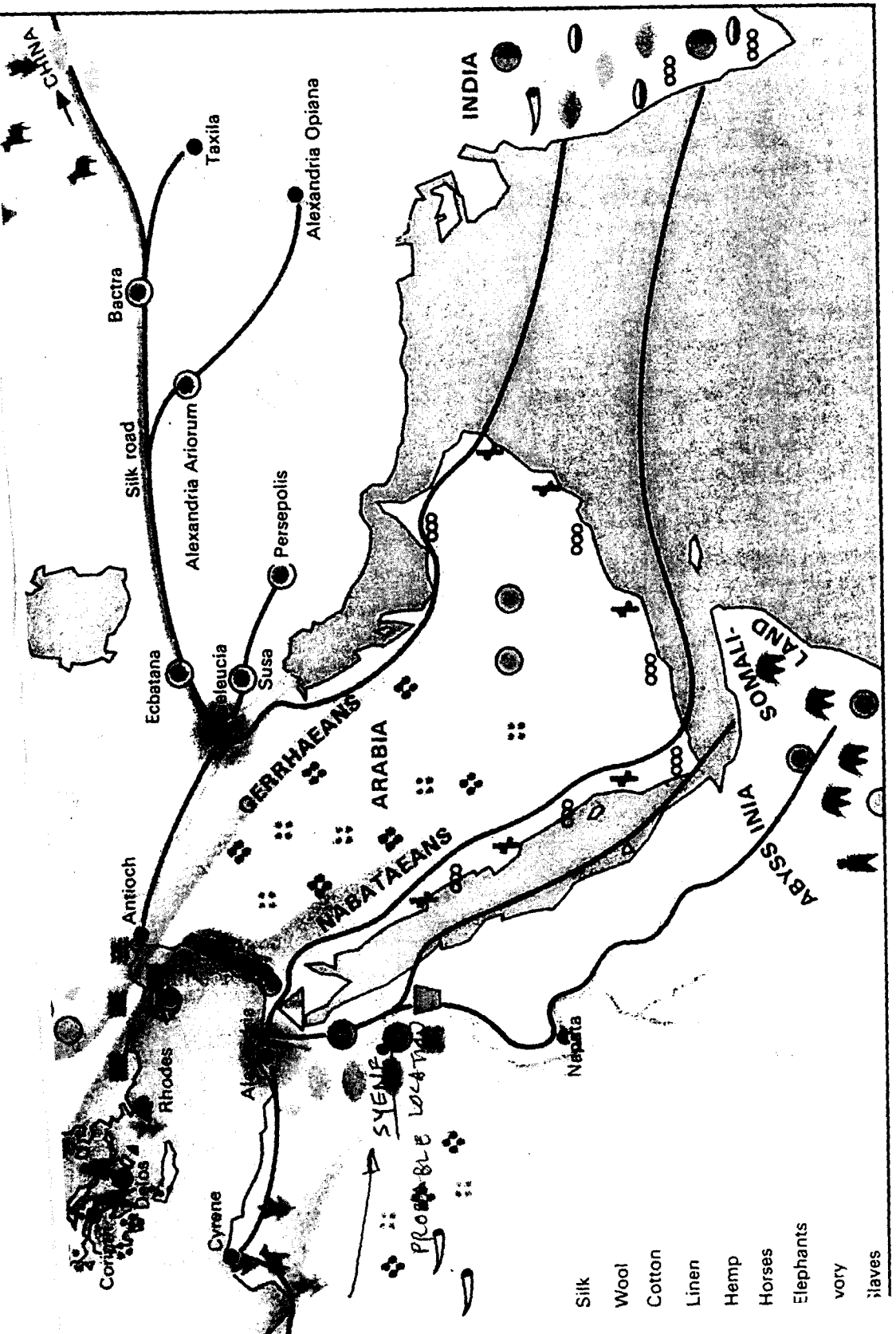
The Hellenistic Period/Economic Life, Cultural Life



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